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INTERNAL KINETIC TRANSFERS IN THE DISPERSED PHASE OF A LOW CONCENTRATION SUSPENSION FLOW

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Abstract—A theoretical analysis of a dilute fluid–solid suspension flow, based on the derivation of equations for linear and angular momentum fluxes in the solid phase, is presented. These equations turn out to be particularly useful in the case of solid particles which only respond very slightly to fluid turbulent fluctuations, when the flow is bounded by solid walls. The applications of the method to a simple example of two-dimensional steady-state flow demonstrates the possibility of a second-order closure. Modelling and the expression of boundary conditions are obtained from the study of particle–wall collisions. In this way it is possible to establish a set of algebraic relations between kinetic transfers in the solid phase.

Key Words: suspension flow, gas–solid, pneumatic transport, particles

1. INTRODUCTION

Among the suspension flows of solid particles, which belong to the two-phase flow family, some are characterized by the important influence of the walls. The most typical example is provided by pneumatic conveying, which is also the oldest application since it dates from the last century. In this sort of flow, the average aero- or hydrodynamic forces and the gravity forces have a preponderant role to play, particularly if the particles have a sufficiently high inertia to prevent their trajectory being influenced to any significant extent by turbulent fluid fluctuations. As a consequence, particle–wall collisions often occur far more frequently than particle–particle collisions. Flow in which particle–particle collisions have a negligible role to play are known as low concentration flows or dilute suspension flows. In this case, the presence of solid walls, combined with the fact that the initial conditions are different for each particle, leads to a disorganized motion of the solid phase: the trajectories of the particles passing through the same point are all different. This is enough to cause internal linear momentum transfers, and possibly angular momentum transfers, in the solid phase.

To be more precise, we know that a local theoretical approach for a fluid–solid flow implies that we write averaged equations expressing, for each of the phases, the conservation of mass, momentum and energy (if the physical characteristics of the fluid are constant, as in our case, this last equation is needless for studying the dynamic problem). The averaging process causes the appearance, in the dispersed phase equations of motion, of terms analogous to Reynolds stresses, whose analyses, which are based on a "diffusion" type argument, can be satisfactory in some cases but do not fit the circumstances in others.

The expression of these momentum transfers by means of a solid phase diffusivity (or "viscosity") has above all been used to deal with pipe flows of suspensions involving relatively small particles. Thus, Soo (1969) and Soo & Tung (1971), like Duckworth & Chan (1976), paid particular attention to the influence of gravity and electrostatic forces. Nagarajan & Murgatroyd (1971) obtained relative velocities and concentration distributions in the absence of gravity and electrostatic effects. Kramer & Depew (1972) calculated the velocity profiles for the case of uniform concentration in a vertical pipe. Williams & Crane (1981) studied the possibility of deposition, under the influence of turbulence, for small particles of about 10 μm. An analysis based on the mixing length hypothesis has been suggested by Choi & Chung (1983), for a suspension of very fine particles which respond perfectly to the turbulent fluid fluctuations. Outside the influence of turbulence, it is also possible to talk about diffusion when the solid phase concentration is high enough to result in a large number of particle–particle collisions.

On the other hand, diffusion style arguments are not appropriate if we are concerned with a dilute suspension flow which is controlled mostly by the average fluid motion, i.e. the inertia of the particles is so high that their motion is unaffected by the turbulence: in this case, one cannot talk about diffusion as being a major mechanism of the transfer phenomena, since the trajectory of a particle is not subjected to fluctuations. In these circumstances, often encountered in gas–solids flows, we can ask what is the origin of the existence of these momentum fluxes such as Reynolds stresses. Is it possible to ignore these terms, as has often been done? Except for special cases, the answer is no, because these "stresses" are related to differences which can exist, both in magnitude and direction, between the velocity vectors of particles located in the same volume element. These differences exist as soon as we are confronted with a disorganized motion due to particle–wall collisions. On this point we agree with Crowe (1982), who indicates that the "shear stress" term for the particle cloud is often neglected, even though "there is presently no quantitative justification for excluding this term, especially near a boundary where the particle can exchange momentum with the wall". It should be emphasized that the hypothesis of a dilute suspension, which implies no interactions between particles, is not sufficient for cancelling these momentum transfers, unlike the assumption made by Rietema & van den Akker (1983).

All the particles contained in the volume element must have the same velocity if these "stresses" are to vanish. This hypothesis can be considered to be implicitly admitted in the studies (carried out in the absence of gravity) of Hamed & Tabakoff (1973), who solve a non-steady flow along a flat plate, of Crooke & Walsh (1974), who deal with a two-dimensional flow in a pipe of infinite length, and of Mitra & Bhattacharyya (1982), who consider non-steady flow between parallel plates. Durst *et al.* (1984) deal with a vertical pipe flow, starting with simplified equations which do not contain "shear stress" terms for the solid phase: this approximation results in a practically uniform relative velocity profile, which is physically unreasonable. The same simplified equations are used by Laitone (1981) to solve a gas–solid flow round a cylinder, and by Di Giacinto *et al.* (1982) for flow in a convergent channel. In these last two cases it seems to us that nothing authorizes the exclusion of the momentum transfer terms in the dispersed phase.

These momentum transfers are influenced by fluid–particle interactions, the resultant of which can possibly depend on the rotational velocity of the particle: this is generally the case for the lift force. Therefore, variations in the average angular velocity in the solid phase have to be taken into account. This means we have to write not only the linear momentum conservation but also the angular momentum conservation in the solid phase. This permits us to express the total momentum of the particles as a function of the velocities of their centres, by translating the fact that the velocities of different points of the same particle are connected due to its non-deformability. Then, after averaging, we will see correlations appear (second-order moments) between the linear and angular velocity components. These terms express internal angular momentum transfer in the solid phase. The set of "stresses" mentioned above and these angular momentum transfers will be termed "internal kinetic transfers" in the dispersed phase.

In this paper, we propose a study of these kinetic transfers in a dilute suspension flow, based on the use of the equations of motion for individual particles, in order to compensate for the loss of information caused by the averaging process. The general formulation presented is not simply limited to flows for which diffusion models are inadequate, but is also applicable for cases with particles which respond well to turbulent fluctuations. The starting point is, of course, provided by the equations of motion of the solid phase. These equations include the conservation of angular momentum, which, to make matters as simple as possible, will only be written for the case of particles possessing a spherically symmetrical inertia tensor. Having deduced equations verified by linear and angular momentum fluxes, we will point out the possibility of a second-order closure, using a simple example where particles are assumed to react only very slightly to turbulent fluid fluctuations. In this case, the study of particle–wall collisions can aid in the modelling and obtaining of boundary conditions.

2. EQUATIONS OF MOTION OF THE SOLID PHASE

Here, we have to apply the general equations which govern a multiphase flow to the case where one of the phases is made up of solid particles in a suspension. Drew (1983) established such equations through a general averaging process, which includes the normal space and time averages as special cases. These equations are in keeping with the instantaneous space-averaged equations of, for example, Fortier (1967), Nigmatulin (1979), Prosperetti & Jones (1984) or Kataoka (1986).

In the following section, equations are derived for the conservation of linear and angular momentum, and it is necessary, for the simplicity of the latter, to assume that the particles are identical. That is the reason why a simple, non-mass-weighted average could be selected but it must be emphasized that this is not the only way to model the problem.

2.1. Instantaneous space-averaged equations

Let us write as $\langle q \rangle$ the instantaneous space phase average of any property q of the solid phase, namely

$$\langle q \rangle = \frac{1}{V_s}\int_{V_s} q \, \mathrm{d}V_s, \qquad [1]$$

where V_s is the volume of the solid phase located in the volume V where the averaging operation takes place, and let

$$\epsilon = \frac{V_s}{V} \qquad [2]$$

be the volume fraction of the solid phase.

The instantaneous equations of continuity and momentum conservation for the solid phase may be written as

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial}{\partial x_k}(\epsilon \langle u_k \rangle) = 0 \qquad [3]$$

and

$$\frac{\partial}{\partial t}(\epsilon \langle u_i \rangle) + \frac{\partial}{\partial x_k}(\epsilon \langle u_k u_i \rangle) = -\frac{\epsilon}{\rho_s}\frac{\partial \tilde{p}}{\partial x_i} + \epsilon \langle f_i \rangle + \epsilon g_i, \qquad [4]$$

where u_i ($i = 1, 2, 3$) are the velocity components, $\tilde{p}$ is the mean pressure in the volume element V, ρ_s is the solid material density (assumed to be constant) and g_i is the x_i-component of the acceleration of gravity. $\langle f_i \rangle$ denotes the space average of the actions exerted by the fluid on the particles, per unit mass and projection onto x_i. In this way it groups the drag and lift forces acting on the N particles whose mass centre belongs to the volume V, assuming that each of the N particles is entirely contained within this volume, i.e. that no particle is cut by the boundary of the volume element. This is possible in a dilute suspension if the shape of the volume V is chosen judiciously (see Rietema & van den Akker 1983).

By introducing, for any quantity q, the deviation Δq, defined by

$$\Delta q = q - \langle q \rangle \quad \text{(and therefore } \langle \Delta q \rangle = 0\text{)}, \qquad [5]$$

[4] can be transformed with the help of the continuity equation [3] in order to obtain the following form, which contains explicitly the stress terms for the solid phase:

$$\epsilon\left(\frac{\partial \langle u_i \rangle}{\partial t} + \langle u_k \rangle \frac{\partial \langle u_i \rangle}{\partial x_k}\right) = -\frac{\epsilon}{\rho_s}\frac{\partial \tilde{p}}{\partial x_i} + \epsilon \langle f_i \rangle + \epsilon g_i - \frac{\partial}{\partial x_k}(\epsilon \langle \Delta u_k \Delta u_i \rangle). \qquad [6]$$

Notice that when all the particles have the same volume, the space average $\langle \mathbf{u} \rangle$ of the velocity vector is equal to the arithmetical mean of the velocities of their mass centres. Indeed, let $\mathbf{u}^p$ and V_s^p denote, respectively, the velocity of the mass centre and the volume of the particle "p". As the density ρ_s is assumed to be constant, we obtain

$$\langle \mathbf{u} \rangle = \frac{1}{V_s}\int_{V_s} \mathbf{u} \, \mathrm{d}V_s = \frac{1}{V_s}\sum_{p=1}^{N}\int_{V_s^p} \mathbf{u} \, \mathrm{d}V_s^p = \frac{1}{V_s}\sum_{p=1}^{N} V_s^p \mathbf{u}^p \qquad [7]$$

and, if $V_s = NV_s^p$,

$$\langle \mathbf{u} \rangle = \frac{1}{N}\sum_{p=1}^{N} \mathbf{u}^p. \qquad [8]$$

The deviation $\Delta\mathbf{u}^p$ can then be interpreted as the difference between the velocity (of the mass centre) of the particle "p" and the space-averaged velocity of the solid phase.

For the sake of simplicity, especially for expressing the conservation of angular momentum, which is necessary for taking variations in the mean angular velocity into account, we will retain, in our following argument, the hypothesis of identical particles. We will also assume the particles are spherical (or at least have a spherically symmetrical inertia tensor), so that the angular momentum relative to the centre of mass of a particle rotating with angular velocity $\boldsymbol{\omega}$ is expressed simply by $J\boldsymbol{\omega}$, J being the moment of inertia of the particle with respect to any axis passing through its centre.

The angular momentum of a single particle, relative to the centre of mass G of the N particles contained in the volume V, can be expressed by

$$J\boldsymbol{\omega}^p + m\mathbf{GG}^p \times \mathbf{u}^p,$$

where $\mathbf{G}^p$ is the centre of mass of the particle. Averaging this expression over the N particles, one obtains the mean angular momentum in the volume element V:

$$\langle J\boldsymbol{\omega}^p + m\mathbf{GG}^p \times \mathbf{u}^p\rangle = J\langle\boldsymbol{\omega}\rangle + m\langle\mathbf{GG}^p \times \mathbf{u}^p\rangle \simeq J\langle\boldsymbol{\omega}\rangle \qquad [9]$$

since the average of the second term on the l.h.s. is of an order of magnitude lower than $J\langle\boldsymbol{\omega}\rangle$ when the components of $\mathbf{GG}^p$ approach zero.

This result is quite analogous with the average linear momentum. Thus, taking the convective transfers into account, and using the same reasoning as that which lead to the averaged momentum equations, we can now obtain the equations expressing the conservation of angular momentum:

$$\frac{\partial}{\partial t}(\epsilon\langle\omega_i\rangle) + \frac{\partial}{\partial x_k}(\epsilon\langle u_k\omega_i\rangle) = \epsilon\frac{\langle M_i\rangle}{J}. \qquad [10]$$

A more straightforward derivation of [4] and [10] will be given at the beginning of section 3, by direct averaging of the equations of motion of a single particle.

After decomposition, [10] can be written as

$$\epsilon\left(\frac{\partial\langle\omega_i\rangle}{\partial t} + \langle u_k\rangle\frac{\partial\langle\omega_i\rangle}{\partial x_k}\right) = \epsilon\frac{\langle M_i\rangle}{J} - \frac{\partial}{\partial x_k}(\epsilon\langle\Delta u_k\Delta\omega_i\rangle). \qquad [11]$$

In these equations, M_i denotes the x_i-component of the moment, at the centre of a particle, of the actions exerted by the fluid on the particles. The last term on the r.h.s. of [11] represents the divergence of an angular momentum flux, due to the existence of the deviations $\Delta\mathbf{u}$ and $\Delta\boldsymbol{\omega}$, which are not independent.

As far as we know, the only authors interested in the conservation of angular momentum in the solid phase are Hamed & Tabakoff (1973). Unfortunately, the equation they obtained (for two-dimensional flows) implies the absence of angular momentum transfers that we have just shown to exist, and whose effects are certainly as important as those produced by linear momentum transfers, providing that transversal forces are present due to rotational motion.

The last terms on the r.h.s. of [6] and [11] show that the momentum transfers in the solid phase are related to the differences between the velocities of individual particles—this is a very important point, justifying the decomposition proposed in [5].

2.2. Time-averaged equations of motion

Even in the absence of any significant influence of the turbulence on the motion of the particles, the volume fraction ϵ together with the space averages of the various kinematic quantities are subject to fluctuations due to the stochastic nature of such a flow. These fluctuations will produce additional momentum and angular momentum transfers, which can only be formulated by performing a time-averaging process. Although this will lead to a more complicated equations, it is necessary to introduce a new decomposition in order to take the influence of these time fluctuations into account. Subsequently, we will denote by an overbar a time-average calculation by integration over a long time interval compared to the time scale of these fluctuations, and we will identify the deviations between instantaneous and average values with a prime. Therefore, we will write

$$\epsilon = \bar{\epsilon} + \epsilon' \qquad [12]$$

and

$$\langle u_i \rangle = \overline{\langle u_i \rangle} + \langle u_i \rangle'. \qquad [13]$$

We can see that this corresponds, for the quantity u_i, to a double decomposition since [13] is equivalent to

$$u_i = \overline{\langle u_i \rangle} + u_i'' \qquad [14]$$

with

$$u_i'' = \Delta u_i + \langle u_i \rangle', \qquad [15]$$

and consequently,

$$\langle u_i'' \rangle = \langle u_i \rangle' \qquad [16]$$

and

$$\overline{\langle u_i'' \rangle} = 0. \qquad [17]$$

The basic difference between the local instantaneous equations already presented and the time-averaged equations lies in the existence of supplementary terms resulting from fluctuations in the concentration of the solid phase. Otherwise, the momentum equations remain formally identical, the terms $\epsilon \langle \Delta u_k \Delta u_i \rangle$ and $\epsilon \langle \Delta u_k \Delta \omega_i \rangle$, which are representative of kinetic transfers, being replaced by $\overline{\epsilon \langle u_k'' u_i'' \rangle}$ and $\overline{\epsilon \langle u_k'' \omega_i'' \rangle}$. Indeed, the continuity equation transforms into

$$\frac{\partial \bar{\epsilon}}{\partial t} + \frac{\partial}{\partial x_k}(\bar{\epsilon}\, \overline{\langle u_k \rangle}) = -\frac{\partial}{\partial x_k} \overline{\epsilon' \langle u_k \rangle'}, \qquad [18]$$

whereas the momentum equations can be time-averaged, bearing in mind [18] and the identity

$$\overline{\epsilon \langle u_k u_i \rangle} = \bar{\epsilon}\, \overline{\langle u_k \rangle}\, \overline{\langle u_i \rangle} + \overline{\langle u_i \rangle}\, \overline{\epsilon' \langle u_k \rangle'} + \overline{\langle u_k \rangle}\, \overline{\epsilon' \langle u_i \rangle'} + \overline{\epsilon \langle u_k'' u_i'' \rangle}, \qquad [19]$$

to obtain

$$\begin{aligned} \bar{\epsilon}\left(\frac{\partial \overline{\langle u_i \rangle}}{\partial t} + \overline{\langle u_k \rangle} \frac{\partial \overline{\langle u_i \rangle}}{\partial x_k}\right) = {} & -\frac{1}{\rho_s}\left(\bar{\epsilon} \frac{\partial \bar{p}}{\partial x_i} + \overline{\epsilon' \frac{\partial \bar{p}'}{\partial x_i}}\right) + \bar{\epsilon} \overline{\langle f_i \rangle} + \overline{\epsilon' \langle f_i \rangle'} + \bar{\epsilon} g_i \\ & - \frac{\partial}{\partial t} \overline{\epsilon' \langle u_i \rangle'} - \overline{\langle u_k \rangle} \frac{\partial}{\partial x_k} \overline{\epsilon' \langle u_i \rangle'} \\ & - \overline{\epsilon' \langle u_i \rangle'} \frac{\partial \overline{\langle u_k \rangle}}{\partial x_k} - \overline{\epsilon' \langle u_k \rangle'} \frac{\partial \overline{\langle u_i \rangle}}{\partial x_k} - \frac{\partial}{\partial x_k} \overline{\epsilon \langle u_k'' u_i'' \rangle}. \end{aligned} \qquad [20]$$

The same operation can be performed with the conservation of angular momentum. This leads to

$$\begin{aligned} \bar{\epsilon}\left(\frac{\partial \overline{\langle \omega_i \rangle}}{\partial t} + \overline{\langle u_k \rangle} \frac{\partial \overline{\langle \omega_i \rangle}}{\partial x_k}\right) = {} & \frac{1}{J}(\overline{\bar{\epsilon} \langle M_i \rangle} + \overline{\epsilon' \langle M_i \rangle'}) - \frac{\partial}{\partial t} \overline{\epsilon' \langle \omega_i \rangle'} - \overline{\langle u_k \rangle} \frac{\partial}{\partial x_k} \overline{\epsilon' \langle \omega_i \rangle'} \\ & - \overline{\epsilon' \langle \omega_i \rangle'} \frac{\partial \overline{\langle u_k \rangle}}{\partial x_k} - \overline{\epsilon' \langle u_k \rangle'} \frac{\partial \overline{\langle \omega_i \rangle}}{\partial x_k} - \frac{\partial}{\partial x_k} \overline{\epsilon \langle u''_k \omega_i'' \rangle}. \end{aligned} \qquad [21]$$

Equations [18], [20] and [21] are just as applicable for a flow in which the dispersed phase movement is influenced by fluid turbulence as for a flow of highly inert particles which have "smooth" trajectories, because this does not prevent the existence of time fluctuations of the space-averaged values. Notice, however, that in the case of a dilute suspension, i.e. where there is no particle interaction, velocity fluctuations in the solid phase cannot be related to concentration fluctuations unless the trajectory of a particle depends on the mean distance between it and the others. This can happen in the case of small particles via fluctuations in the fluid velocity, possibly influenced by the concentration, which in turn influence the particles' velocity fluctuations. It follows that with the hypothesis of a dilute suspension flow of highly inert particles, the only terms which are influenced by concentration fluctuations are the pressure term in [20] and the terms $\overline{\epsilon'\langle f_i\rangle'}$ and $\overline{\epsilon'\langle M_i\rangle'}$, which are related to fluid velocity fluctuations.

In the following section, we will establish, with the only restriction being that of a dilute suspension, the transport equations for average kinetic transfers $\overline{\epsilon\langle u_k'' u_i''\rangle}$ and $\overline{\epsilon\langle u_k''\omega_i''\rangle}$, allowing us to envisage a second-order closure of the momentum equations [20] and [21]. For the application described in section 4, we will adopt the further hypothesis of independence between concentration fluctuations and velocity fluctuations, as mentioned above. This assumption, which applies to the case of particles with high inertia, results in considerable simplification of the various equations.

3. KINETIC TRANSFER EQUATIONS

In order to establish these equations, we must perform the averaging operation of the substantial derivative (following the motion of a particle) of any extensive property q of the particles. This operation obeys an identity which is analogous to the classical transport theorem for a fluid, and was given by Nigmatulin (1979) for the general case of a multiphase flow. When applied to a solid particle suspension (with constant ρ_s), this "transport theorem" can be written as

$$\left\langle\frac{\mathrm{d}q}{\mathrm{d}t}\right\rangle = \frac{1}{\epsilon}\left[\frac{\partial}{\partial t}(\epsilon\langle q\rangle) + \frac{\partial}{\partial x_k}(\epsilon\langle u_k q\rangle)\right]. \qquad [22]$$

This equation is a generalization of a result which was derived by Fortier (1967), in the case where q = momentum per unit mass, by introducing a probability density function for the particle velocity and examining the rates of momentum in and out through the boundary of the control volume.

Notice that, when $q = u_i$, [22] can be applied to the averaging of the individual equation of motion of a particle,

$$\frac{\mathrm{d}u_i}{\mathrm{d}t} = -\frac{1}{\rho_s}\frac{\partial\tilde{p}}{\partial x_i} + f_i + g_i, \qquad [23]$$

leading to [4]. The same thing is true for the angular momentum conservation equation:

$$\frac{\mathrm{d}\omega_i}{\mathrm{d}t} = \frac{M_i}{J}, \qquad [24]$$

which yields [10], after averaging and application of [22] with $q = \omega_i$.

The equation of motion [23], multiplied by u_j, then averaged, leads to

$$\left\langle u_j\frac{\mathrm{d}u_i}{\mathrm{d}t}\right\rangle = -\frac{1}{\rho_s}\langle u_j\rangle\frac{\partial\tilde{p}}{dx_i} + \langle u_j f_i\rangle + \langle u_j\rangle g_i \qquad [25]$$

which can be added to the x_j-component of the equation of motion, multiplied by u_i, in order to obtain an expression for the averaged time derivative of the product $u_i u_j$:

$$\left\langle\frac{\mathrm{d}(u_i u_j)}{\mathrm{d}t}\right\rangle = -\frac{1}{\rho_s}\left(\langle u_i\rangle\frac{\partial\tilde{p}}{\partial x_j} + \langle u_j\rangle\frac{\partial\tilde{p}}{\partial x_i}\right) + \langle u_i f_j\rangle + \langle u_j f_i\rangle + \langle u_i\rangle g_j + \langle u_j\rangle g_i. \qquad [26]$$

Application of [22] yields:

$$\frac{\partial}{\partial t}(\epsilon\langle u_i u_j\rangle) + \frac{\partial}{\partial x_k}(\epsilon\langle u_i u_j u_k\rangle) = -\frac{\epsilon}{\rho_s}\left(\langle u_i\rangle\frac{\partial\tilde{p}}{\partial x_j} + \langle u_j\rangle\frac{\partial\tilde{p}}{\partial x_i}\right) + \epsilon\langle u_i f_j\rangle + \epsilon\langle u_j f_i\rangle + \epsilon\langle u_i\rangle g_j + \epsilon\langle u_j\rangle g_i \qquad [27]$$

which results, after transformation and time-averaging, in a system of six partial differential equations for the correlations $\overline{\epsilon\langle u_i'' u_j''\rangle}$. This transformation is somewhat tedious and so it is worth describing just the main features. The averages $\overline{\epsilon\langle u_i u_j\rangle}$ and $\overline{\epsilon\langle u_i u_j u_k\rangle}$ are expressed using the following identities, which are easy to obtain:

$$\overline{\epsilon\langle u_i u_j\rangle} = \overline{\langle u_i\rangle}\,\overline{\epsilon\langle u_j\rangle} + \overline{\langle u_j\rangle}\,\overline{\epsilon\langle u_i\rangle} - \bar{\epsilon}\,\overline{\langle u_i\rangle}\,\overline{\langle u_j\rangle} + \overline{\epsilon\langle u_i'' u_j''\rangle} \qquad [28]$$

and

$$\begin{aligned}\overline{\epsilon\langle u_iu_ju_k\rangle} &= \overline{\langle u_i\rangle}\,\overline{\epsilon\langle u_ju_k\rangle} + \overline{\langle u_j\rangle}\,\overline{\epsilon\langle u_iu_k\rangle} - \overline{\epsilon\langle u_k\rangle}\,\overline{\langle u_i\rangle}\,\overline{\langle u_j\rangle} \\ &\quad + \overline{\langle u_k\rangle}\,\overline{\epsilon\langle u_i''u_j''\rangle} + \overline{\epsilon\langle u_i''u_j''u_k''\rangle}. \end{aligned} \qquad [29]$$

The l.h.s. of [27] is expressed with the help of [28] and [29]. After rearranging, one obtains

$$\begin{aligned}\overline{\frac{\partial}{\partial t}(\epsilon\langle u_iu_j\rangle)} + \overline{\frac{\partial}{\partial x_k}(\epsilon\langle u_iu_ju_k\rangle)} &= \frac{\partial}{\partial t}\overline{\epsilon\langle u_i''u_j''\rangle} + \overline{\langle u_k\rangle}\frac{\partial}{\partial x_k}\overline{\epsilon\langle u_i''u_j''\rangle} \\ &\quad + \overline{\epsilon\langle u_i''u_j''\rangle}\frac{\partial\overline{\langle u_k\rangle}}{\partial x_k} + \overline{\langle u_i\rangle}\left(\overline{\frac{\partial}{\partial t}(\epsilon\langle u_j\rangle) + \frac{\partial}{\partial x_k}(\epsilon\langle u_ku_j\rangle)}\right) \\ &\quad + \overline{\langle u_j\rangle}\left(\overline{\frac{\partial}{\partial t}(\epsilon\langle u_i\rangle) + \frac{\partial}{\partial x_k}(\epsilon\langle u_ku_i\rangle)}\right) \\ &\quad + \overline{\epsilon'\langle u_i\rangle'}\left(\frac{\partial\overline{\langle u_j\rangle}}{\partial t} + \overline{\langle u_k\rangle}\frac{\partial\overline{\langle u_j\rangle}}{\partial x_k}\right) \\ &\quad + \overline{\epsilon'\langle u_j\rangle'}\left(\frac{\partial\overline{\langle u_i\rangle}}{\partial t} + \overline{\langle u_k\rangle}\frac{\partial\overline{\langle u_i\rangle}}{\partial x_k}\right) \\ &\quad + \overline{\epsilon\langle u_i''u_k''\rangle}\frac{\partial\overline{\langle u_j\rangle}}{\partial x_k} + \overline{\epsilon\langle u_j''u_k''\rangle}\frac{\partial\overline{\langle u_i\rangle}}{\partial x_k} + \frac{\partial}{\partial x_k}(\overline{\epsilon\langle u_i''u_j''u_k''\rangle}). \end{aligned} \qquad [30]$$

The l.h.s.s of the x_i- and x_j-components of the instantaneous momentum equation [4], which appear in the above equation, are then replaced by the corresponding r.h.s.s. After simplification and use of continuity equation [3], all we have to do is identify with [27]. Finally, it is found that [27] can be written in the following form, which is characteristic of transport equations for the correlations $\overline{\epsilon\langle u_i''u_j''\rangle}$ since they concern their substantial derivatives calculated by following the mean average motion of the solid phase:

$$\begin{aligned}\frac{\partial}{\partial t}\overline{\epsilon\langle u_i''u_j''\rangle} + \overline{\langle u_k\rangle}\frac{\partial}{\partial x_k}\overline{\epsilon\langle u_i''u_j''\rangle} &= \overline{\epsilon'\langle u_i\rangle'}\,\overline{\langle f_j\rangle} + \overline{\epsilon'\langle u_j\rangle'}\,\overline{\langle f_i\rangle} + \overline{\epsilon\langle u_i''f_j''\rangle} + \overline{\epsilon\langle u_j''f_i''\rangle} \\ &\quad + \overline{\epsilon'\langle u_i\rangle'}g_j + \overline{\epsilon'\langle u_j\rangle'}g_i - \overline{\epsilon'\langle u_i\rangle'}\left(\frac{\partial\overline{\langle u_j\rangle}}{\partial t} + \overline{\langle u_k\rangle}\frac{\partial\overline{\langle u_j\rangle}}{\partial x_k}\right) \\ &\quad - \overline{\epsilon'\langle u_j\rangle'}\left(\frac{\partial\overline{\langle u_i\rangle}}{\partial t} + \overline{\langle u_k\rangle}\frac{\partial\overline{\langle u_i\rangle}}{\partial x_k}\right) - \overline{\epsilon\langle u_i''u_k''\rangle}\frac{\partial\overline{\langle u_j\rangle}}{\partial x_k} \\ &\quad - \overline{\epsilon'\langle u_j''u_k''\rangle}\frac{\partial\overline{\langle u_i\rangle}}{\partial x_k} - \overline{\epsilon\langle u_i''u_j''\rangle}\frac{\partial\overline{\langle u_k\rangle}}{\partial x_k} - \frac{\partial}{\partial x_k}\overline{\epsilon\langle u_i''u_j''u_k''\rangle} \\ &\quad - \frac{1}{\rho_s}\left[\bar{\epsilon}\left(\overline{\langle u_i\rangle'\frac{\partial\tilde{p}'}{\partial x_j}} + \overline{\langle u_j\rangle'\frac{\partial\tilde{p}'}{\partial x_i}}\right) + \overline{\epsilon'\langle u_i\rangle'\frac{\partial\tilde{p}}{\partial x_j}} + \overline{\epsilon'\langle u_j\rangle'\frac{\partial\tilde{p}}{\partial x_i}}\right]. \end{aligned} \qquad [31]$$

Apart from terms involving concentration fluctuations, the new unknowns which we must therefore try to model are the velocity–pressure correlations, the correlations introduced by the interaction terms $\overline{\epsilon\langle u_i''f_j''\rangle}$ and $\overline{\epsilon\langle u_j''f_i''\rangle}$, as well as terms $\overline{\epsilon\langle u_i''u_j''u_k''\rangle}$.

To establish the transport equations for correlations $\overline{\epsilon\langle u_i''\omega_j''\rangle}$, we will follow the same procedure: we multiply [23], relative to u_i, by ω_j, and add this to the angular momentum equation [24], relative to ω_j, multiplied by u_i. Thus we arrive, after using the transport theorem, at

$$\overline{\frac{\partial}{\partial t}(\epsilon\langle u_i\omega_j\rangle)} + \overline{\frac{\partial}{\partial x_k}(\epsilon\langle u_ku_i\omega_j\rangle)} = -\overline{\frac{\epsilon}{\rho_s}\langle\omega_j\rangle\frac{\partial\tilde{p}}{\partial x_i}} + \overline{\epsilon\langle\omega_jf_i\rangle} + \overline{\epsilon\langle\omega_j\rangle}g_i + \overline{\frac{\epsilon}{J}\langle u_iM_j\rangle} \qquad [32]$$

that is to say a set of nine equations for the nine correlations $\overline{\epsilon\langle u_i\omega_j\rangle}$, or $\overline{\epsilon\langle u_i''\omega_j''\rangle}$, since the averages $\overline{\epsilon\langle u_i\omega_j\rangle}$ and $\overline{\epsilon\langle u_k u_i\omega_j\rangle}$ can be expressed using [28] and [29] to obtain transport equations of correlations $\overline{\epsilon\langle u_i''\omega_j''\rangle}$ in a form comparable to [31]:

$$\begin{aligned}\frac{\partial}{\partial t}\overline{\epsilon\langle u_i''\omega_j''\rangle} + \overline{\langle u_k\rangle}\frac{\partial}{\partial x_k}\overline{\epsilon\langle u_i''\omega_j''\rangle} = &-\frac{1}{\rho_s}\left(\bar{\epsilon}\,\overline{\langle\omega_j\rangle'\frac{\partial\tilde{p}'}{\partial x_i}} + \overline{\epsilon'\langle\omega_j\rangle'\frac{\partial\tilde{p}}{\partial x_i}}\right) + \overline{\epsilon'\langle\omega_j\rangle'}g_i\\ &+\overline{\epsilon'\langle\omega_j\rangle'}\,\overline{\langle f_i\rangle} + \overline{\epsilon\langle\omega_j''f_i''\rangle}\\ &+\frac{1}{J}\left(\overline{\epsilon'\langle u_i\rangle'}\,\overline{\langle M_j\rangle} + \overline{\epsilon\langle u_i''M_j''\rangle}\right)\\ &-\overline{\epsilon'\langle u_i\rangle'}\left(\frac{\partial\overline{\langle\omega_j\rangle}}{\partial t} + \overline{\langle u_k\rangle}\frac{\partial\overline{\langle\omega_j\rangle}}{\partial x_k}\right)\\ &-\overline{\epsilon'\langle\omega_j\rangle'}\left(\frac{\partial\overline{\langle u_i\rangle}}{\partial t} + \overline{\langle u_k\rangle}\frac{\partial\overline{\langle u_i\rangle}}{\partial x_k}\right)\\ &-\overline{\epsilon\langle u_i''u_k''\rangle}\frac{\partial\overline{\langle\omega_j\rangle}}{\partial x_k} - \overline{\epsilon\langle\omega_j''u_k''\rangle}\frac{\partial\overline{\langle u_i\rangle}}{\partial x_k}\\ &-\overline{\epsilon\langle u_i''\omega_j''\rangle}\frac{\partial\overline{\langle u_k\rangle}}{\partial x_k} - \frac{\partial}{\partial x_k}\overline{\epsilon\langle u_k''u_i''\omega_j''\rangle}.\end{aligned} \qquad [33]$$

The six second-order correlations $\overline{\epsilon\langle\omega_i''\omega_j''\rangle}$ can figure on the r.h.s. of these equations, via terms $\overline{\epsilon\langle\omega_j''f_i''\rangle}$, because the action of the fluid on a particle is generally a function of its angular velocity. These moments have not appeared up to now either in [6] and [11] or in [31]. This is why it might possibly be necessary to write, in addition to [31] and [33], the equations for the second-order correlations $\overline{\epsilon\langle\omega_i''\omega_j''\rangle}$. These are obtained by multiplying each of the angular momentum equations [24] by each of the angular velocity components ω_j, i.e.

$$\left\langle\omega_j\frac{\mathrm{d}\omega_i}{\mathrm{d}t}\right\rangle = \frac{1}{J}\langle\omega_j M_i\rangle \qquad [34]$$

then, adding this to the analogous equation giving $\left\langle\omega_i\frac{\mathrm{d}\omega_j}{\mathrm{d}t}\right\rangle$:

$$\left\langle\frac{\mathrm{d}(\omega_i\omega_j)}{\mathrm{d}t}\right\rangle = \frac{1}{J}(\langle\omega_i M_j\rangle + \langle\omega_j M_i\rangle); \qquad [35]$$

or, with the transport theorem [22],

$$\frac{\partial}{\partial t}(\epsilon\langle\omega_i\omega_j\rangle) + \frac{\partial}{\partial x_k}(\epsilon\langle\omega_i\omega_j u_k\rangle) = \frac{\epsilon}{J}(\langle\omega_i M_j\rangle + \langle\omega_j M_i\rangle). \qquad [36]$$

These six new equations can be transformed as before to be written as

$$\begin{aligned}\frac{\partial}{\partial t}\overline{\epsilon\langle\omega_i''\omega_j''\rangle} + \overline{\langle u_k\rangle}\frac{\partial}{\partial x_k}\overline{\epsilon\langle\omega_i''\omega_j''\rangle} = &\frac{1}{J}\left(\overline{\epsilon'\langle\omega_i\rangle'}\,\overline{\langle M_j\rangle} + \overline{\epsilon'\langle\omega_j\rangle'}\,\overline{\langle M_i\rangle} + \overline{\epsilon\langle\omega_i''M_j''\rangle} + \epsilon(\omega_j''M_i'')\right)\\ &-\overline{\epsilon'\langle\omega_i\rangle'}\left(\frac{\partial\overline{\langle\omega_j\rangle}}{\partial t} + \overline{\langle u_k\rangle}\frac{\partial\overline{\langle\omega_j\rangle}}{\partial x_k}\right)\\ &-\overline{\epsilon'\langle\omega_j\rangle'}\left(\frac{\partial\overline{\langle\omega_i\rangle}}{\partial t} + \overline{\langle u_k\rangle}\frac{\partial\overline{\langle\omega_i\rangle}}{\partial x_k}\right)\\ &-\overline{\epsilon\langle\omega_i''u_k''\rangle}\frac{\partial\overline{\langle\omega_j\rangle}}{\partial x_k} - \overline{\epsilon\langle\omega_j''u_k''\rangle}\frac{\partial\overline{\langle\omega_i\rangle}}{\partial x_k}\\ &-\overline{\epsilon\langle\omega_i''\omega_j''\rangle}\frac{\partial\overline{\langle u_k\rangle}}{\partial x_k} - \frac{\partial}{\partial x_k}\overline{\epsilon\langle\omega_i''\omega_j''u_k''\rangle}.\end{aligned} \qquad [37]$$

Equations [18], [20], [21], [31], [33] and [37] make up the system which has to be solved if we wish to use a second-order model to obtain, for example, profiles of translational velocities, rotational velocities or concentration of the dispersed phase.

We have not mentioned the equations governing the fluid phase motion, which are coupled to the preceding ones via interaction terms; however, we know that the dilute suspension hypothesis allows us to neglect the action of the solid phase on the fluid phase, and therefore to solve the flow of the latter independently from that of the solid phase ("one-way" coupling). Experimental results concerning gas–solid pipe flows, obtained for instance by Tsuji & Morikawa (1982), Tsuji *et al.* (1984) and Oesterlé (1987a), confirm that the fluid velocity profile is not significantly altered by the presence of particles provided that the loading ratio is $\lesssim 0.1$. Nevertheless, this assumption is not absolutely necessary in the following, where the fluid velocity $\mathbf{v}$ appears: all the derivations in section 4 remain valid, even if the fluid velocity is unknown, but in this case the solid phase equations have to be coupled with the fluid phase equations, thus leading to a much more complicated problem. Whatever the hypothesis concerning this point, the general problem is in any event an extremely complex one, and so for the moment we are obliged to limit ourselves to a simple case. The example dealt with in the following section, the purpose of which is to demonstrate the ways the theory can be applied in practice, is of a steady-state two-dimensional flow of a dilute suspension of high inertia spherical particles.

4. APPLICATION TO A SUSPENSION OF COARSE PARTICLES

Our definition of "coarse particles" applies to high inertia particles, which do not respond significantly to turbulent fluid fluctuations. We have already indicated that in this case there is independence between concentration fluctuations and velocity fluctuations (instantaneous space averages) of the solid phase. This leads to the vanishing of ϵ'-terms in the foregoing equations, and allows the replacement of time averages $\overline{\epsilon\langle u_i'' u_j''\rangle}$, $\overline{\epsilon\langle u_i''\omega_j''\rangle}$ or $\overline{\epsilon\langle\omega_i''\omega_j''\rangle}$, respectively, by $\bar{\epsilon}\overline{\langle u_i'' u_j''\rangle}$, $\bar{\epsilon}\overline{\langle u_i''\omega_j''\rangle}$ or $\bar{\epsilon}\overline{\langle\omega_i''\omega_j''\rangle}$.

This hypothesis is generally unsuited to liquid–solid flows. Therefore we prefer to deal with a gas–solid flow, where we know, in addition, that the effect of the pressure gradient on the motion of the particles is negligible, as pointed out by Soo (1982).

We will also limit ourselves to a plane two-dimensional configuration, where the only linear velocity components are u_x and u_y, and where the only angular velocity component is ω_z. Thus, we are led to a system of simplified equations, whose closure requires modelling of the third-order correlations $\overline{\langle u_x''^3\rangle}$, $\overline{\langle u_x''^2 u_y''\rangle}$, $\overline{\langle u_x'' u_y''^2\rangle}$, $\overline{\langle u_y''^3\rangle}$, $\overline{\langle u_x''^2\omega_z''\rangle}$, $\overline{\langle u_x'' u_y''\omega_z''\rangle}$, $\overline{\langle u_y''^2\omega_z''\rangle}$, $\overline{\langle u_x''\omega_z''^2\rangle}$ and $\overline{\langle u_y''\omega_z''^2\rangle}$. Useful information on this subject can be obtained by studying the behaviour of the solid phase in the neighbourhood of the wall. Indeed, as indicated above, the existence of kinetic transfers in this kind of flow is related to particle–wall collisions, and we can therefore expect that the whole of the flow will be dominated by this phenomenon.

Next, the interaction terms will be described in detail, and finally we will show how the system of kinetic transfer equations can lead, under certain conditions, to their formulation by algebraic expressions.

4.1. Study in the neighbourhood of a wall

This study is based on the results concerning a particle–wall collision. We will suppose the wall to be normal to direction y, as in figure 1. The velocity components just before impact will be designated by a subscript 1, those just after impact by a subscript 2. We will call a the radius of the spherical particle, μ_0 and μ the static and dynamic friction factors and e the coefficient of restitution, defined by

$$u_{y2} = -e u_{y1}. \qquad [38]$$

It can be easily shown that the collision occurs without sliding if

$$|u_{x1} - a\omega_{z1}| < \tfrac{7}{2}\mu_0(1+e)u_{y1}. \qquad [39]$$

In this case, the velocity components u_x and ω_z after impact are

$$u_{x2} = \tfrac{1}{7}(5u_{x1} + 2a\omega_{z1}) \qquad [40a]$$

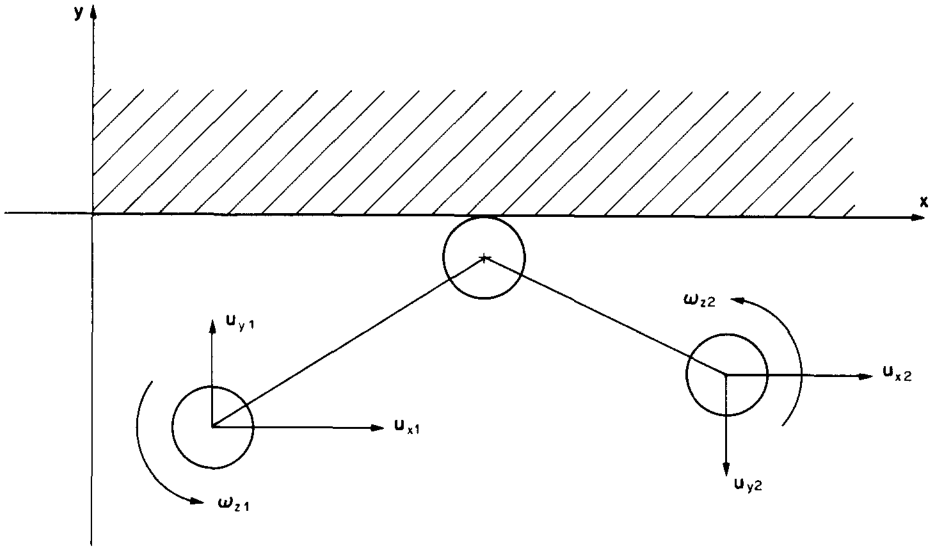

Figure 1. Particle–wall collision configuration.

and

$$\omega_{z2} = \frac{u_{x2}}{a}. \qquad [40b]$$

In a suspension flow along a wall, with $u_{x1} > 0$, the sliding velocity at the beginning of the collision, which is $u_{x1} - a\omega_{z1}$, is generally positive and much higher than u_{y1}. Therefore inequality [39] cannot be satisfied, so we are mainly concerned here with collisions accompanied by sliding. In this case, we obtain

$$u_{x2} = u_{x1} - \mu(1+e)u_{y1} \qquad [41a]$$

and

$$\omega_{z2} = \omega_{z1} + \tfrac{5}{2}\mu(1+e)\frac{u_{y1}}{a}. \qquad [41b]$$

These relations can be used to estimate the relative values of the various correlations at the wall. Among the N particles occupying a volume element adjoining the wall, let N_1 be the number of particles moving towards the wall ("p_1" particles) and N_2 the number of particles moving away from it ("p_2" particles). We will introduce the notations $\langle q \rangle_1$ and $\langle q \rangle_2$ to designate the average values calculated for these two classes of particles:

$$\langle q \rangle_1 = \frac{1}{N_1}\sum_{p_1=1}^{N_1} q^{(p_1)}, \quad \langle q \rangle_2 = \frac{1}{N_2}\sum_{p_2=1}^{N_2} q^{(p_2)}. \qquad [42]$$

The condition of zero mass flux at the wall is written as

$$N_1\langle u_y \rangle_1 + N_2\langle u_y \rangle_2 = 0. \qquad [43]$$

Relations [38] and [41], valid for a particle, also express approximately the average velocity components of the "p_2" particles in terms of the average velocity components of the "p_1" particles. This would be entirely exact if we take an extreme model, where all the "p_1" particles have the same linear and angular velocity. In this hypothesis, the definition of the coefficient of restitution leads to

$$\langle u_y \rangle_2 \approx -e\langle u_y \rangle_1 \qquad [44]$$

and

$$\langle u_y^2 \rangle_2 \approx e^2\langle u_y^2 \rangle_1. \qquad [45]$$

These relations, together with [43], are also valid for time-averages. Thus, we deduce from [43] and [44] the expression of the average value at the wall of any quantity q:

$$\langle q\rangle_w = \frac{\overline{N_1}\,\overline{\langle q\rangle_1} + \overline{N_2}\,\overline{\langle q\rangle_2}}{\overline{N_1} + \overline{N_2}} \approx \frac{e\overline{\langle q\rangle_1} + \overline{\langle q\rangle_2}}{e+1}. \qquad [46]$$

Notice immediately that from [44]–[46] we can estimate the quadratic average $\overline{\langle u_y^2\rangle_w}$, and hence $\overline{\langle u_y''^2\rangle_w}$ (since $\overline{\langle u_y\rangle_w} = 0$), as a function of the average normal velocity of particles moving towards the wall:

$$\overline{\langle u_y''^2\rangle_w} = \overline{\langle u_y^2\rangle_w} \approx e\overline{\langle u_y^2\rangle_1} \approx e\overline{\langle u_y\rangle_1}^2, \qquad [47]$$

whereas relations [41a, b] lead to

$$\overline{\langle u_x u_y\rangle_2} \approx -e\overline{\langle u_x u_y\rangle_1} + \mu e(1+e)\overline{\langle u_y^2\rangle_1} \qquad [48a]$$

and

$$\overline{\langle u_y \omega_z\rangle_2} \approx -e\overline{\langle u_y \omega_z\rangle_1} - \frac{5\mu}{2a}e(1+e)\overline{\langle u_y^2\rangle_1}. \qquad [48b]$$

By applying [46] and bearing in mind $\langle u_y\rangle_w = 0$ (which implies $\langle u_x u_y\rangle_w = \langle u_x'' u_y''\rangle_w$ and $\langle u_y \omega_z\rangle_w = \langle u_y'' \omega_z''\rangle_w$), we obtain

$$\overline{\langle u_x'' u_y''\rangle_w} \approx \mu\overline{\langle u_y''^2\rangle_w} \qquad [49a]$$

and

$$\overline{\langle u_y'' \omega_z''\rangle_w} \approx -\frac{5\mu}{2a}\overline{\langle u_y''^2\rangle_w}. \qquad [49b]$$

For the other second-order correlations, we use identities such as

$$\overline{\langle u_i'' u_j''\rangle} = \overline{\langle u_i u_j\rangle} - \overline{\langle u_i\rangle}\,\overline{\langle u_j\rangle}, \qquad [50]$$

which result in

$$\overline{\langle u_x''^2\rangle_w} \approx \mu^2\overline{\langle u_y''^2\rangle_w}, \qquad [51a]$$

$$\overline{\langle u_x'' \omega_z''\rangle_w} \approx -\frac{5\mu^2}{2a}\overline{\langle u_y''^2\rangle_w} \qquad [51b]$$

and

$$\overline{\langle \omega_z''^2\rangle_w} \approx \frac{25\mu^2}{4a^2}\overline{\langle u_y''^2\rangle_w} \qquad [51c]$$

Although these results are based on a simplifying schematization, they plainly indicate that second-order correlations representative of kinetic transfers cannot be supposed to be non-existent in the neighbourhood of a wall. Additionally, the foregoing relations provide boundary conditions which are possibly of use in solving equations governing the solid phase flow.

The same process, applied to third-order correlations, with the help of averaging identities such as

$$\overline{\langle u_i'' u_j'' u_k''\rangle} = \overline{\langle u_i u_j u_k\rangle} - \overline{\langle u_i\rangle}\,\overline{\langle u_j\rangle}\,\overline{\langle u_k\rangle} - \overline{\langle u_i\rangle}\,\overline{\langle u_j'' u_k''\rangle} - \overline{\langle u_j\rangle}\,\overline{\langle u_k'' u_i''\rangle} - \overline{\langle u_k\rangle}\,\overline{\langle u_i'' u_j''\rangle}, \qquad [52]$$

can yield, after some calculations, the following estimations:

$$\overline{\langle u_x''^3\rangle_w} \approx \mu^3\overline{\langle u_y''^3\rangle_w}, \qquad [53a]$$

$$\overline{\langle u_x''^2 u_y''\rangle_w} \approx \mu^2\overline{\langle u_y''^3\rangle_w}, \qquad [53b]$$

$$\overline{\langle u_x'' u_y''^2\rangle_w} \approx \mu\overline{\langle u_y''^3\rangle_w}, \qquad [53c]$$

$$\overline{\langle u_x''^2 \omega_z''\rangle_w} \approx -\frac{5\mu^3}{2a}\overline{\langle u_y''^3\rangle_w}, \qquad [53d]$$

$$\overline{\langle u_x'' u_y'' \omega_z''\rangle_w} \approx \frac{5\mu^2}{2a}\overline{\langle u_y''^3\rangle_w}, \qquad [53e]$$

$$\overline{\langle u_y''^2 \omega_z'' \rangle_w} \approx -\frac{5\mu}{2a} \overline{\langle u_y''^3 \rangle_w}, \qquad [53f]$$

$$\overline{\langle u_x'' \omega_z''^2 \rangle_w} \approx \frac{25\mu^3}{4a^2} \overline{\langle u_y''^3 \rangle_w} \qquad [53g]$$

and

$$\overline{\langle u_y'' \omega_z''^2 \rangle_w} \approx \frac{25\mu^2}{4a^2} \overline{\langle u_y''^3 \rangle_w}. \qquad [53h]$$

In these relations [53a–h], the various third-order correlations are expressed as a function of one of them, namely $\overline{\langle u_y''^3 \rangle_w}$, which can be approximated, thanks to [46], by

$$\overline{\langle u_y''^3 \rangle_w} \approx e(1-e)\overline{\langle u_y^3 \rangle_1} \qquad [54]$$

or, in terms of $\overline{\langle u_y''^2 \rangle_w}$, according to [47]:

$$\overline{\langle u_y''^3 \rangle_w} \approx \frac{1-e}{\sqrt{e}} \sqrt{\overline{\langle u_y''^2 \rangle_w}}^{\,3}. \qquad [55]$$

Relations [53a–h], when associated with [55], can constitute the basis of the modelling of the third-order correlations, in so far as all of these are expressed in terms of the second-order correlation $\overline{\langle u_y''^2 \rangle}$. Although these relationships were derived in the neighbourhood of the wall, it is reasonable to apply them to the remainder of the flow field, since the whole flow is governed by this particle–wall collision mechanism.

However, in order to avoid unnecessary complexity at this stage, we will later consider these third-order moments to be negligible. This hypothesis is based on the following argument. Equation [55] makes possible the comparison between the orders of magnitude of correlations such as $\overline{\langle u_i'' u_j'' u_k'' \rangle}$ and terms such as $\overline{\langle u_i \rangle}\, \overline{\langle u_j'' u_k'' \rangle}$. For example, $\overline{\langle u_x''^3 \rangle}$ must be compared with $\overline{\langle u_x \rangle}\, \overline{\langle u_x''^2 \rangle}$. Equations [53a] and [55] lead to

$$\overline{\langle u_x''^3 \rangle_w} \approx \mu^3 \frac{1-e}{\sqrt{e}} \sqrt{\overline{\langle u_y''^2 \rangle_w}}^{\,3}, \qquad [56]$$

whereas, according to [51a]

$$\overline{\langle u_x \rangle_w}\, \overline{\langle u_x''^2 \rangle_w} \approx \mu^2 \overline{\langle u_x \rangle_w}\, \overline{\langle u_y''^2 \rangle_w}. \qquad [57]$$

In the most frequent case of a restitution coefficient not too far from unity, the r.h.s. coefficient in [56] is small compared to unity, so there is no doubt that

$$\overline{\langle u_x \rangle_w} \gg \mu \frac{1-e}{\sqrt{e}} \sqrt{\overline{\langle u_y''^2 \rangle_w}} \qquad [58]$$

and we can indeed see that we do have, at least in the neighbourhood of the wall and probably in all the flow field

$$\overline{\langle u_x''^3 \rangle} \ll \overline{\langle u_x \rangle}\, \overline{\langle u_x''^2 \rangle}. \qquad [59]$$

The same argument applies to the other third-order correlations and leads to similar results, thus justifying the hypothesis made in the closure example advanced later in this paper.

4.2. Expression of the interaction terms

At this stage, all we have left to do in order to close the system made up of the equations of motion and the equations for kinetic transfers (or second-order moment equations) is to express the correlations between velocity fluctuations and fluctuations in aerodynamic forces, in terms of the first- or second-order moments.

First of all, let us examine the terms related to fluctuations in the aerodynamic resultant, which consists of the drag force, which we will call $\mathbf{f}_D$ (per unit mass), and the lift force, $\mathbf{f}_L$. It is always possible to define the relaxation time t^* of a particle by expressing the drag force per unit mass as

$$\mathbf{f}_D = -\frac{1}{t^*}(\mathbf{u} - \mathbf{v}), \qquad [60]$$

where $\mathbf{v}$ is the fluid velocity ("at a distance" from the particle). For a gas–solid flow (i.e. $\rho_s \gg \rho_f$, ρ_f being the fluid density), and if Stoke's law applies, the relaxation time is given by

$$t^* = \frac{2\rho_s a^2}{9\eta}, \qquad [61]$$

where η is the dynamic viscosity of the fluid.

This supposes that the Reynolds number (based on relative velocity) of the sphere is much smaller than unity. If this is not the case, the relaxation time, which can always be defined by [60], is not constant, but is a function of the relative velocity magnitude. Therefore, in order to express velocity–drag correlations in a usable form, it is convenient to assign locally a constant value to t^*, corresponding to an average relaxation time for the particles located in the same volume element. The only aim of this hypothesis is to allow us to make the following approximation:

$$\left\langle \frac{1}{t^*} u_i'' u_j'' \right\rangle \simeq \frac{1}{t^*} \langle u_i'' u_j'' \rangle \qquad [62]$$

which will be useful in the derivation of the velocity–drag correlations with respect to the second-order velocity correlations.

How this approximation is suitable in the case of coarse particles, for which the Reynolds number based on the relative velocity is often higher than unity, can be evaluated by the following numerical example. Consider glass beads, with density $\rho_s = 2600\,\mathrm{kg/m^3}$ and dia 0.1 mm, conveyed by air in a 20 mm dia pipe, with velocity ~ 30 m/s: it was shown (Oesterlé 1986) that in such a gas–solid flow, the motion of the particles is not influenced by the fluid turbulence, while the mean relative velocity is close to 0.6 m/s, corresponding to a mean Reynolds number close to 4. At this Reynolds number, the actual relaxation time is $t^* = 0.060$ s. If the relative velocity is reduced to 0.3 m/s or increased to 1.2 m/s, then the actual relaxation time will be increased to 0.067 s or reduced to 0.051 s, respectively. It is believed that such variations are sufficiently slight to permit us to make the assumption defined by [62].

Lift, which is due to the rotation of the particle relative to the fluid, was calculated, the Reynolds number still being small, by Rubinow & Keller (1961). Per unit mass, this lift can be written as

$$\mathbf{f}_L = \frac{3}{4}\frac{\rho_f}{\rho_s}\left(\boldsymbol{\omega} - \frac{1}{2}\nabla \times \mathbf{v}\right) \times (\mathbf{u} - \mathbf{v}). \qquad [63]$$

Although experimental data are lacking at present in the range of intermediate relative Reynolds numbers, this expression seems to be valid for Reynolds numbers up to 10, since it was applied by White & Schulz (1977) for the computation of particle trajectories, with relative Reynolds numbers of the order of 10, leading to an excellent agreement with experimentally observed trajectories.

It must be pointed out that lift can exist, due solely to the velocity gradient, without any rotational velocity. This was calculated by Saffman (1965), but it is very small compared with that in the case of high inertia particles. Therefore, we will confine ourselves to expression [63] which, connected with [60] and [62], leads to the following results for the correlations between velocity and aerodynamic resultant components:

$$\overline{\langle u_x'' f_x'' \rangle} = -\frac{1}{t^*}\overline{\langle u_x''^2 \rangle} - \frac{3\rho_f}{4\rho_s}\left\{\left[\overline{\langle \omega_z \rangle} + \frac{1}{2}\left(\frac{\partial \overline{v}_x}{\partial y} - \frac{\partial \overline{v}_y}{\partial x}\right)\right]\overline{\langle u_x'' u_y'' \rangle} + (\overline{\langle u_y \rangle} - \overline{v}_y)\overline{\langle u_x'' \omega_z'' \rangle}\right\}, \qquad [64a]$$

$$\overline{\langle u_y'' f_x'' \rangle} = -\frac{1}{t^*}\overline{\langle u_x'' u_y'' \rangle} - \frac{3\rho_f}{4\rho_s}\left\{\left[\overline{\langle \omega_z \rangle} + \frac{1}{2}\left(\frac{\partial \overline{v}_x}{\partial y} - \frac{\partial \overline{v}_y}{\partial x}\right)\right]\overline{\langle u_y''^2 \rangle} + (\overline{\langle u_y \rangle} - \overline{v}_y)\overline{\langle u_y'' \omega_z'' \rangle}\right\}, \qquad [64b]$$

$$\overline{\langle u_x'' f_y'' \rangle} = -\frac{1}{t^*}\overline{\langle u_x'' u_y'' \rangle} + \frac{3\rho_f}{4\rho_s}\left\{\left[\overline{\langle \omega_z \rangle} + \frac{1}{2}\left(\frac{\partial \overline{v}_x}{\partial y} - \frac{\partial \overline{v}_y}{\partial x}\right)\right]\overline{\langle u_x''^2 \rangle} + (\overline{\langle u_x'' \rangle} - \overline{v}_x)\overline{\langle u_x'' \omega_z'' \rangle}\right\}, \qquad [64c]$$

$$\overline{\langle u_y'' f_y'' \rangle} = -\frac{1}{t^*}\overline{\langle u_y''^2 \rangle} + \frac{3\rho_f}{4\rho_s}\left\{\left[\overline{\langle \omega_z \rangle} + \frac{1}{2}\left(\frac{\partial \overline{v}_x}{\partial y} - \frac{\partial \overline{v}_y}{\partial x}\right)\right]\overline{\langle u_x'' u_y'' \rangle} + (\overline{\langle u_x \rangle} - \overline{v}_x)\overline{\langle u_y'' \omega_z'' \rangle}\right\}, \qquad [64d]$$

$$\overline{\langle \omega_z'' f_x'' \rangle} = -\frac{1}{t^*}\overline{\langle u_x''\omega_z''\rangle} - \frac{3\rho_f}{4\rho_s}\left\{\left[\overline{\langle \omega_z\rangle} + \frac{1}{2}\left(\frac{\partial \overline{v_x}}{\partial y} + \frac{\partial \overline{v_y}}{\partial x}\right)\right]\overline{\langle u_y''\omega_z''\rangle} + (\overline{\langle u_y\rangle} - \overline{v_y})\overline{\langle \omega_z''^2\rangle}\right\} \quad [64e]$$

and

$$\overline{\langle \omega_z'' f_y'' \rangle} = -\frac{1}{t^*}\overline{\langle u_y''\omega_z''\rangle} + \frac{3\rho_f}{4\rho_s}\left\{\left[\overline{\langle \omega_z\rangle} + \frac{1}{2}\left(\frac{\partial \overline{v_x}}{\partial y} - \frac{\partial \overline{v_y}}{\partial x}\right)\right]\overline{\langle u_x''\omega_z''\rangle} + (\overline{\langle u_x\rangle} - \overline{v_x})\overline{\langle \omega_z''^2\rangle}\right\}. \quad [64f]$$

These results are rather simple, because fluid velocity turbulent fluctuations and solid phase velocity fluctuations are independent for high inertia particles.

Together with [63], Rubinow & Keller (1961) obtained the theoretical expression of the moment, at the centre of the sphere, of the actions exerted by the fluid on the particle:

$$\mathbf{M} = -8\pi\eta a^3(\boldsymbol{\omega} - \tfrac{1}{2}\nabla \times \mathbf{v}); \quad [65]$$

i.e. by introducing, as with drag, a relaxation time t_1^* for the rotation:

$$\frac{1}{J}\mathbf{M} = \frac{1}{t_1^*}(\boldsymbol{\omega} - \tfrac{1}{2}\nabla \times \mathbf{v}), \quad [66]$$

where $J = 2ma^2/5$ is the moment of inertia of the sphere and

$$t_1^* = \frac{\rho_s a^2}{15\eta}. \quad [67]$$

From this we can deduce the expressions for the velocity–torque correlations:

$$\overline{\langle u_x'' M_z''\rangle} = -\frac{J}{t_1^*}\overline{\langle u_x''\omega_z''\rangle}, \quad [68a]$$

$$\overline{\langle u_y'' M_z''\rangle} = -\frac{J}{t_1^*}\overline{\langle u_y''\omega_z''\rangle} \quad [68b]$$

and

$$\overline{\langle \omega_z'' M_z''\rangle} = -\frac{J}{t_1^*}\overline{\langle \omega_z''^2\rangle}. \quad [68c]$$

4.3. Consequences on kinetic transfers in the solid phase

Here we will limit ourselves to establishing a simple formulation, via algebraic relations, of kinetic transfers. This is valid for such a steady-state two-dimensional flow, as the convection terms which appear in the system of equations obtained in section 3 are negligible or non-existent. We can, for example, take the case of a parallel or nearly parallel flow (direction x) in which we can assume $\langle u_y\rangle \simeq 0$, which covers a fairly wide range of practical applications (pipe flows, for instance).

In these circumstances, [31], [33] and [37] lead to the following set of six equations:

$$-\frac{1}{t^*}\overline{\langle u_y''^2\rangle} + \frac{3\rho_f}{4\rho_s}[\overline{\langle u_{xr}\rangle}\,\overline{\langle u_y''\omega_z''\rangle} + \overline{\langle \omega_{zr}\rangle}\,\overline{\langle u_x''u_y''\rangle}] = 0, \quad [69a]$$

$$-\frac{2}{t^*}\overline{\langle u_x''u_y''\rangle} + \frac{3\rho_f}{4\rho_s}[\overline{\langle u_{xr}''\rangle}\,\overline{\langle u_x''\omega_z''\rangle} + \overline{\langle \omega_{zr}\rangle}(\overline{\langle u_x''^2\rangle} - \overline{\langle u_y''^2\rangle})] - \overline{\langle u_y''^2\rangle}\frac{\mathrm{d}\overline{\langle u_x\rangle}}{\mathrm{d}y} = 0, \quad [69b]$$

$$\frac{1}{t^*}\overline{\langle u_x''^2\rangle} + \left(\frac{3\rho_f}{4\rho_s}\overline{\langle \omega_{zr}\rangle} + \frac{\mathrm{d}\overline{\langle u_x\rangle}}{\mathrm{d}y}\right)\overline{\langle u_x''u_y''\rangle} = 0, \quad [69c]$$

$$-\left(\frac{1}{t^*} + \frac{1}{t_1^*}\right)\overline{\langle u_y''\omega_z''\rangle} + \frac{3\rho_f}{4\rho_s}[\overline{\langle u_{xr}\rangle}\,\overline{\langle \omega_z''^2\rangle} + \overline{\langle \omega_{zr}\rangle}\,\overline{\langle u_x''\omega_z''\rangle}] - \overline{\langle u_y''^2\rangle}\frac{\mathrm{d}\overline{\langle \omega_z\rangle}}{\mathrm{d}y} = 0, \quad [69d]$$

$$\left(\frac{1}{t^*} + \frac{1}{t_1^*}\right)\overline{\langle u_x''\omega_z''\rangle} + \left(\frac{3\rho_f}{4\rho_s}\overline{\langle \omega_{zr}\rangle} + \frac{\mathrm{d}\overline{\langle u_x\rangle}}{\mathrm{d}y}\right)\overline{\langle u_y''\omega_z''\rangle} + \overline{\langle u_x''u_y''\rangle}\frac{\mathrm{d}\overline{\langle \omega_z\rangle}}{\mathrm{d}y} = 0 \quad [69e]$$

and

$$\frac{1}{t_1^*}\overline{\langle \omega_z''^2 \rangle} + \overline{\langle u_y'' \omega_z'' \rangle}\frac{\mathrm{d}\overline{\langle \omega_z \rangle}}{\mathrm{d}y} = 0, \quad [69\mathrm{f}]$$

where $\overline{\langle u_{xr} \rangle}$ is the relative linear velocity,

$$\overline{\langle u_{xr} \rangle} = \overline{\langle u_x \rangle} - \overline{v_x}, \quad [70]$$

and $\overline{\langle \omega_{zr} \rangle}$ is the relative angular velocity,

$$\overline{\langle \omega_{zr} \rangle} = \overline{\langle \omega_z \rangle} + \frac{1}{2}\left(\frac{\partial \overline{v_x}}{\partial y} + \frac{\partial \overline{v_y}}{\partial x}\right). \quad [71]$$

Equations [69a–f] form a homogeneous linear system for the six kinetic transfers. Therefore their compatibility supposes that the determinant is zero. After some calculations, we find that this compatibility condition can be translated by the following equation, which expresses a direct relationship between the linear and angular velocity profiles:

$$t^{*2}\overline{\langle \omega_{zr} \rangle}\left(\frac{\mathrm{d}\overline{\langle u_x \rangle}}{\mathrm{d}y} + \frac{3\rho_f}{4\rho_s}\overline{\langle \omega_{zr} \rangle}\right) + t^* t_1^* \overline{\langle u_{xr} \rangle}\frac{\mathrm{d}\overline{\langle \omega_z \rangle}}{\mathrm{d}y} + \frac{4\rho_s}{3\rho_f} = 0. \quad [72]$$

When this condition is satisfied, we can express five unknown second-order correlations in terms of the sixth one, for instance $\overline{\langle u_y''^2 \rangle}$:

$$\overline{\langle u_x''^2 \rangle} = \overline{\langle u_y''^2 \rangle} t^{*2}\left(\frac{\mathrm{d}\overline{\langle u_x \rangle}}{\mathrm{d}y} + \frac{3\rho_f}{4\rho_s}\overline{\langle \omega_{zr} \rangle}\right)^2, \quad [73\mathrm{a}]$$

$$\overline{\langle u_x'' u_y'' \rangle} = -\overline{\langle u_y''^2 \rangle} t^{*}\left(\frac{\mathrm{d}\overline{\langle u_x \rangle}}{\mathrm{d}y} + \frac{3\rho_f}{4\rho_s}\overline{\langle \omega_{zr} \rangle}\right), \quad [73\mathrm{b}]$$

$$\overline{\langle u_x'' \omega_z'' \rangle} = \overline{\langle u_y''^2 \rangle} t^* t_1^* \frac{\mathrm{d}\overline{\langle \omega_z \rangle}}{\mathrm{d}y}\left(\frac{\mathrm{d}\overline{\langle u_x \rangle}}{\mathrm{d}y} + \frac{3\rho_f}{4\rho_s}\overline{\langle \omega_{zr} \rangle}\right), \quad [73\mathrm{c}]$$

$$\overline{\langle u_y'' \omega_z'' \rangle} = -\overline{\langle u_y''^2 \rangle} t_1^* \frac{\mathrm{d}\overline{\langle \omega_z \rangle}}{\mathrm{d}y} \quad [73\mathrm{d}]$$

and

$$\overline{\langle \omega_z''^2 \rangle} = \overline{\langle u_y''^2 \rangle} t_1^{*2}\left(\frac{\mathrm{d}\overline{\langle \omega_z \rangle}}{\mathrm{d}y}\right)^2. \quad [73\mathrm{e}]$$

These algebraic expressions of the second-order moments, representative of kinetic transfers, can be used as shear-stress–deformation-rate relationships for the dispersed solid phase. When combined with the compatibility equation [72] and with the equations of motion, they can lead to a complete solution for a gas–solid flow corresponding to the hypothesis made above.

Furthermore, it should be pointed out that expressions [73a–e] and boundary conditions [49a,b] and [51a–c], which were obtained above by studying particle–wall collisions, are perfectly compatible. From these relations, we can deduce, by identification, the following boundary conditions expressed in terms of the average linear and angular velocities:

$$\left(\frac{\mathrm{d}\overline{\langle u_x \rangle}}{\mathrm{d}y}\right)_{\mathrm{w}} + \frac{3\rho_f}{4\rho_s}\overline{\langle \omega_{zr} \rangle}_{\mathrm{w}} = -\frac{\mu}{t^*} \quad [74]$$

and

$$\left(\frac{\mathrm{d}\overline{\langle \omega_z \rangle}}{\mathrm{d}y}\right)_{\mathrm{w}} = \frac{5\mu}{2at_1^*}. \quad [75]$$

5. CONCLUSIONS

The general equations governing the transfer of linear and angular momentum in the solid phase have been established here for the case of a dilute suspension flow. This formulation is particularly useful for particles which have such high inertia that their motion is barely influenced by fluid turbulence: thus the solid phase flow is only conditioned by the balance between gravity forces, mean aerodynamic actions and kinetic transfers due to the existence of collisions against the solid walls. In these circumstances, we are led to a second-order closure problem, which is solved in section 4 for a simple application, namely a parallel two-dimensional steady-state flow. It seems possible to apply the results of this study to all nearly parallel flows, for example boundary-layer flows, or the flow in the inlet length of a pipe.

The present analysis is based on the use of a non-mass-weighted space average, but it must be emphasized that this is not the only way to model the problem.

To conclude, we would like to point out that the first tests of the calculation of angular velocity profiles are leading to promising results, in so far as they are in good agreement with a numerical simulation being undertaken at the same time (Oesterlé 1986, 1987b).

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